Power Simulations, etc.

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Simple Power Analysis: Which parameters can you vary?

As we saw earlier today, we can make power calculations for varying some of the parameters of the experiment:

- N
- Noise (σ)
- Effect size (τ)

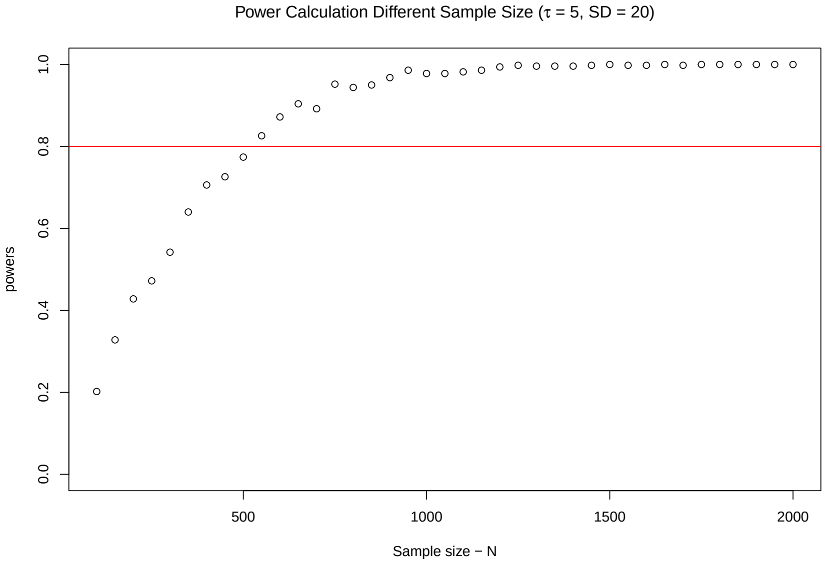
Recall:

$$\text{Power} = \Phi\left(\frac{|\tau|\sqrt{N}}{2\sigma} - \Phi^{-1}(1 - \frac{\alpha}{2})\right)$$

An example of power calculations for different sample size

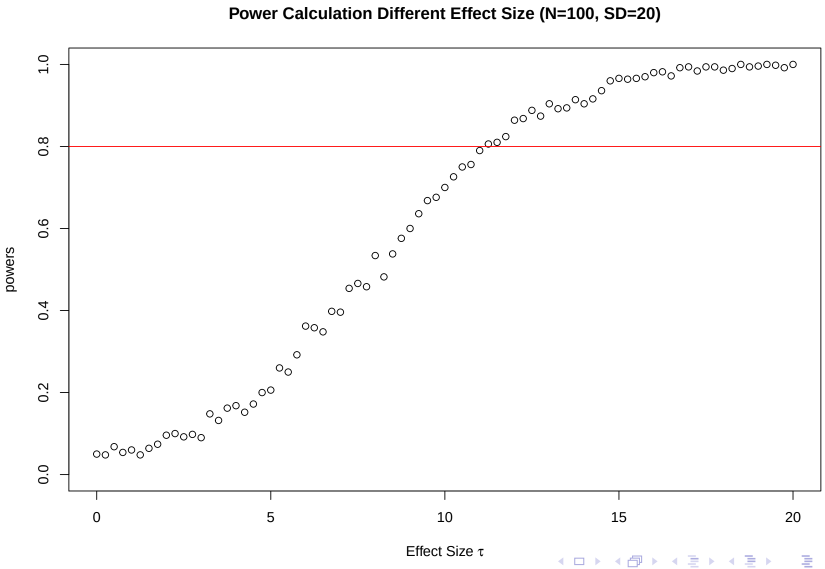
```
possible.ns <- seq(from = 100, to = 2000, by = 50)
powers <- rep(NA, length(possible.ns))
for (j in 1:length(possible.ns)) {
    N <- possible.ns[j]
    significant.experiments <- rep(NA, 500)
    for (i in 1:500) {
        Y0 <- rnorm(n = N, mean = 60, sd = 20)
        tau <- 5
        Y1 <- Y0 + tau
        Z.sim <- rbinom(n = N, size = 1, prob = 0.5)
        Y.sim <- Y1 * Z.sim + Y0 * (1 - Z.sim)
        fit.sim <- lm(Y.sim ~ Z.sim)
        p.value <- summary(fit.sim)$coefficients[2, 4]
        significant.experiments[i] <- (p.value <= 0.05)
    }
    powers[j] <- mean(significant.experiments)
}
```

Let's see how this looks:



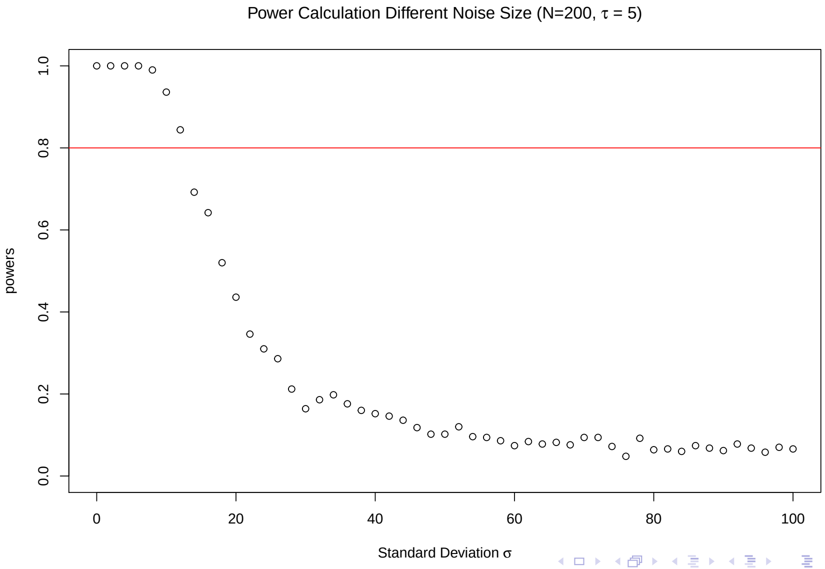
We can also vary other parameters:

1. Let's see what happens with different effect sizes:

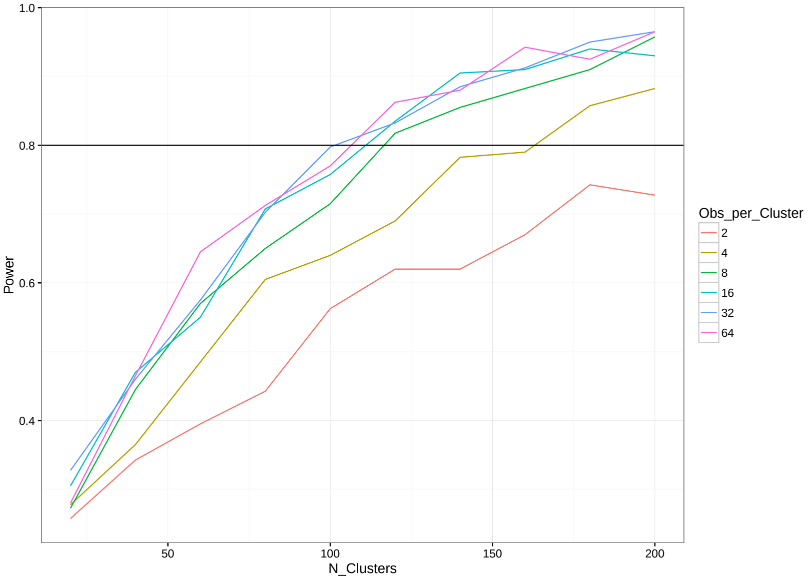


We can also vary other parameters:

2. Let's see what happens with different noise:



Power Analysis for clustered randomized experiments



Factorial Design

Consider the following factorial design:

	$T_2 = 0$	$T_2 = 1$
$T_1 = 0$	$\mathbb{E}[Y\|Z00] = 2$	$\mathbb{E}[Y\|Z01] = 2.5$
$T_1 = 1$	$\mathbb{E}[Y\|Z10] = 2.7$	$\mathbb{E}[Y\|Z11] = 3.7$

- These are standardized effect sizes $\Rightarrow$ unit is standard deviations of $Y|Z00$
- **HUGE** effect sizes in the table above!
- Different estimands here:
  - Marginal effects (many)
  - Conditional marginal effect

Factorial Estimands:

	$T_2 = 0$	$T_2 = 1$
$T_1 = 0$	$\mathbb{E}[Y\|Z00] = 2$	$\mathbb{E}[Y\|Z01] = 2.5$
$T_1 = 1$	$\mathbb{E}[Y\|Z10] = 2.7$	$\mathbb{E}[Y\|Z11] = 3.7$

1. Marginal Effect of $T_1|\ T_2 = 0$:

$$\mathbb{E}[Y|Z10] - \mathbb{E}[Y|Z00] = 2.7 - 2 = 0.7$$

2. Marginal Effect of $T_1|T_2 = 1$:

$$\mathbb{E}[Y|Z11] - \mathbb{E}[Y|Z01] = 3.7 - 2.5 = 1.2$$

3. Marginal Effect of $T_2|T_1 = 0$:

$$\mathbb{E}[Y|Z01] - \mathbb{E}[Y|Z00] = 2.5 - 2 = 0.5$$

4. Marginal Effect of $T_2|T_1 = 1$:

$$\mathbb{E}[Y|Z11] - \mathbb{E}[Y|Z10] = 3.7 - 2.7 = 1$$

Estimands, Continued

	$T_2 = 0$	$T_2 = 1$
$T_1 = 0$	$\mathbb{E}[Y\|Z00] = 2$	$\mathbb{E}[Y\|Z01] = 2.5$
$T_1 = 1$	$\mathbb{E}[Y\|Z10] = 2.7$	$\mathbb{E}[Y\|Z11] = 3.7$

5. Average Marginal Effect of T_1:

$$\frac{1}{2}(\mathbb{E}[Y|Z10] - \mathbb{E}[Y|Z00] + \mathbb{E}[Y|Z11] - \mathbb{E}[Y|Z01]) = .5(.7 + 1.2) = .95$$

6. Average Marginal Effect of T_2:

$$\frac{1}{2}(\mathbb{E}[Y|Z01] - \mathbb{E}[Y|Z00] + \mathbb{E}[Y|Z11] - \mathbb{E}[Y|Z10]) = .5(.5 + 1) = .75$$

7. Conditional Marginal Effect of $T_1|T_2$ (equivalent to CME of $T_2|T_1$):

$$(\mathbb{E}[Y|Z11] - \mathbb{E}[Y|Z10]) - (\mathbb{E}[Y|Z01] - \mathbb{E}[Y|Z00]) = 1 - .5 = .5$$
$$(\mathbb{E}[Y|Z11] - \mathbb{E}[Y|Z01]) - (\mathbb{E}[Y|Z10] - \mathbb{E}[Y|Z00]) = 1.2 - .7 = .5$$

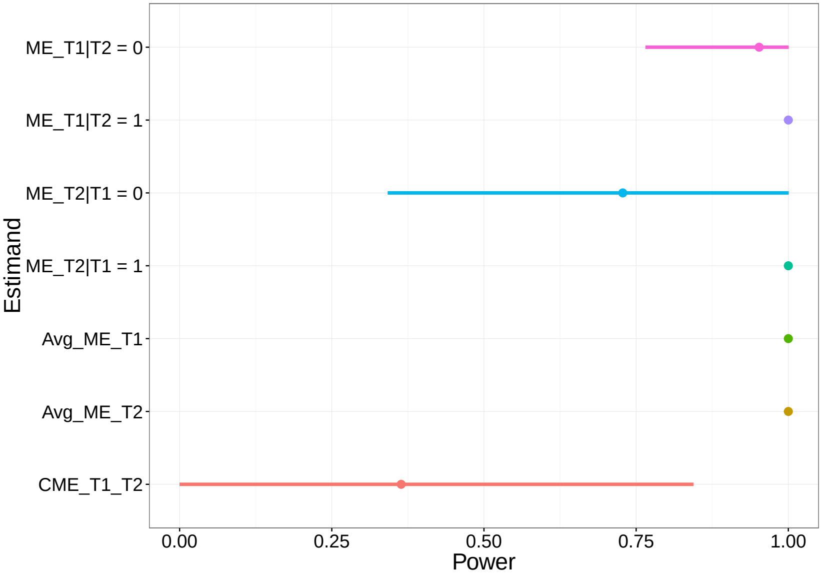
Power Calculation

- Calculated in `DeclareDesign`—we'll learn more tomorrow
- Standard estimator for this 2×2 factorial design is OLS:

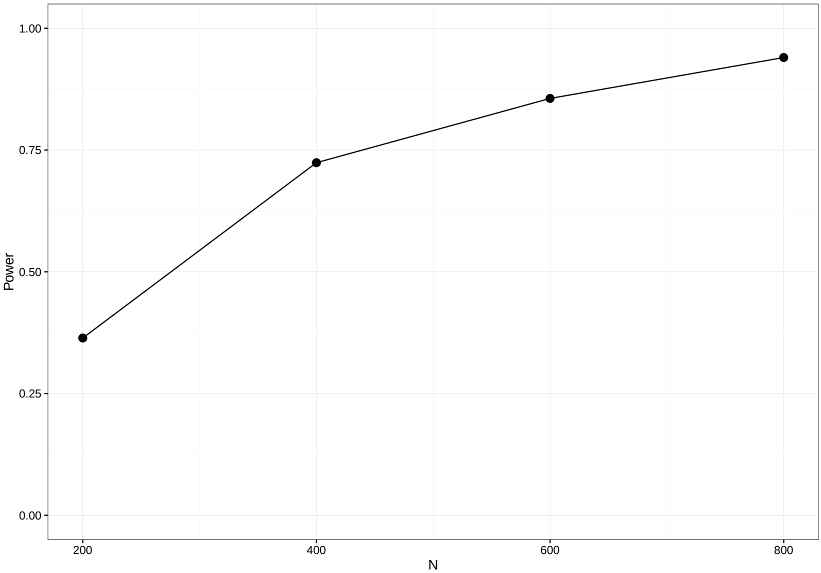
$$y_i = \beta_0 + \beta_1 T_{1i} + \beta_2 T_{2i} + \beta_3 T_{1i} T_{2i} + \epsilon_i$$

- In this case:
  - β_1 estimates Marginal Effect of $T_1 | T_2 = 0$
  - β_2 estimates Marginal Effect of $T_2 | T_1 = 0$
  - β_3 estimates Conditional Marginal Effect of $T_1 | T_2$
- Other effects can be estimated by adding combinations of coefficients
- Assume $N = 200$ and with 50 units in each treatment condition

Power Estimates



Improving Power on CME is Costly



Conclusions: Factorial

- Effect sizes here are **HUGE**
- Generally well-powered for marginal effects – though we may not want this estimand
- Conditional marginal effects – the benefit of factorial
  - Generally very underpowered
  - Consider the costs and benefits of factorial
  - Three arm with T_1 and T_2 may be a preferable design
- Simulation *ex-ante* can help us understand the properties of research designs!